CHAPTER A-5 EVENT TREES

A-5.1 Key Concepts

Event tree analysis is a commonly used tool in dam and levee safety risk analysis to identify, characterize, and estimate risk. Quantitative estimates for probability of failure and the resulting consequences can be obtained using event trees. Graphical depictions of potential failure modes and consequences can also be developed using event trees. Sub trees can be used to further evaluate specific events within the overall event tree structure. Sub trees are typically developed for individual potential failure modes to fully describe the sequence of events and/or conditions required to obtain failure.

A logical progression of events is represented by the event tree beginning with an initiating event and continuing through to a set of outcomes. A typical progression might include an initiating event (flood or earthquake) followed by a system response (breach or non-breach) resulting in potential consequences (life loss, economic). Additional contributing events such as inoperable spillway gates (system response), flood fighting (system response), and exposure (consequences) can also be included in the event tree.

An event tree consists of a sequence of interconnected nodes and branches. Each node is associated with an uncertain event (a crack forms in the embankment) or a state of nature (existence of adversely oriented joint planes). Branches originating from a node represent each of the possible events or states of nature that can occur. Probabilities are estimated for each branch to represent the likelihood for each event or condition. These probabilities are conditional on the occurrence of the preceding events to the left in the tree. Risks are typically annualized (e.g. annual probability of failure [APF] or annual life loss [ALL]) in the event tree by using annual probabilities to characterize the loading conditions. The conditional structure of the event tree allows the probability for any sequence of events to be computed by multiplying the probabilities for each branch along a pathway, in accordance with the multiplication rule of probability theory. The branching structure of the event tree allows the probability for any combination of event sequences (e.g. total failure probability for a potential failure mode) to be computed by summing branch probabilities across multiple pathways.

A-5.2 Event Tree Terminology

An example event tree structure is presented in **Figure A-5-1**. Terms used to describe the event tree structure are illustrated in the figure and defined below.

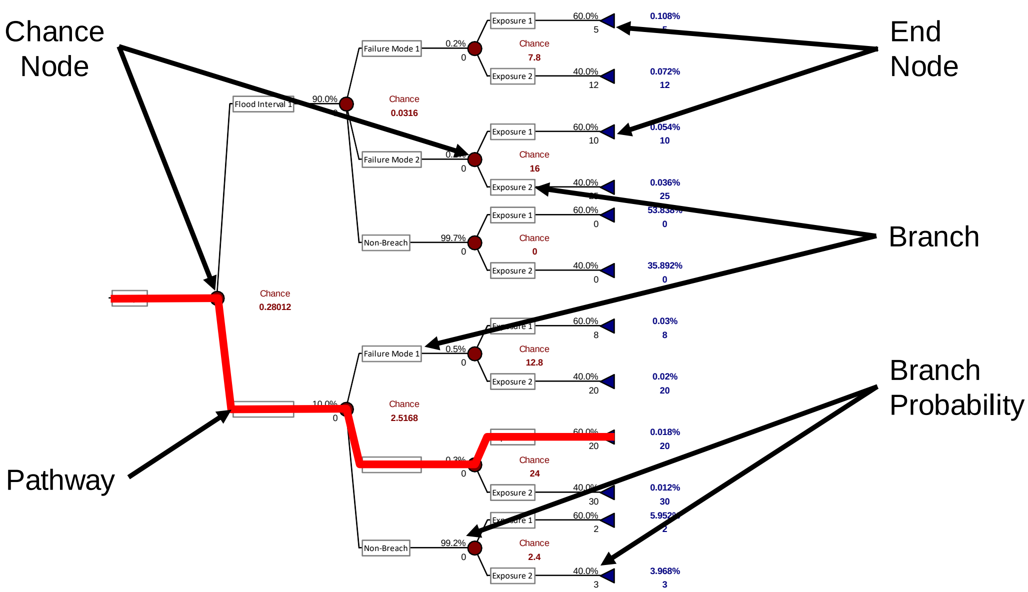


Figure A-5-1 Event Tree Terminology

Branch – Represents an event or outcome and is usually designated by a line segment.

Branch probability – The probability of the event represented by the branch conditioned on the occurrence of the events to its left in the event tree.

Chance Node – A branching point in the event tree usually designated by a circle at the end of a branch indicating the occurrence of an unknown event.

End Node – The outcome of a pathway belonging to the last level of branches in an event tree. An end node defines a possible end state for a sequence of events.

Pathway – A unique sequence of events representing a possible failure progression. Mathematically its probability is represented by the intersection of the events along the pathway.

A-5.3 Event Tree Structure

The starting point for an event tree is a triggering event (or state of nature). For dam and levee risk analysis, this is typically a loading event such as a flood or earthquake. Subsequent events are then defined using a divergent branching structure where each branch represents a unique event. The branching structure is used to define all of the possible events that could occur along a given failure path. The sequencing of events in the tree should be logical but does not necessarily need to be chronological.

Branches and their associated branch probabilities that are statistically dependent on preceding events must be shown along pathways to the right of the events on which they are statistically dependent. This is an important consideration in event tree construction because branch probabilities are mathematically defined as conditional probabilities. This also allows branch probabilities to be a function of a state variable in a preceding branch. The event tree in **Figure A-5-2** illustrates an example where the probability of failure is conditional on obtaining a particular water surface stage during a flood event.

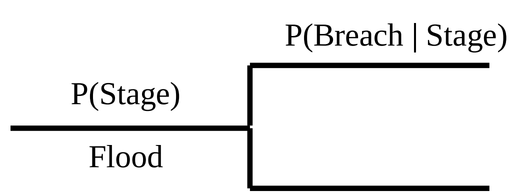


Figure A-5-2 Conditional Event Tree Probaiblity

The conditional structure of the event tree satisfies the probability calculus for the multiplication rule of basic probability theory; therefore, branch probabilities can be multiplied along a pathway to obtain the probability for the intersection of events along the pathway. In the preceding example, the probability of failure can be computed as P(Stage) * P(Breach|Stage). Branches that originate from a chance node are mutually exclusive and collectively exhaustive. This makes each event and pathway unique (mutually exclusive) and ensures that all possible events and pathways are considered (collectively exhaustive). A result of this requirement is that branch probabilities originating from a node can be summed and the total sum across all branches is equal to one. This provides a convenient validation check for the probabilities entered into the event tree, as illustrated in **Figure A-5-3**. In practice, branches might not always be mutually exclusive or collectively exhaustive. For example, events that have a negligible contribution to the risk estimate could be omitted to reduce the size of the event tree. In these cases, validation checks may need to be modified by the risk analyst to account for the structure of the tree being used for a particular risk analysis.

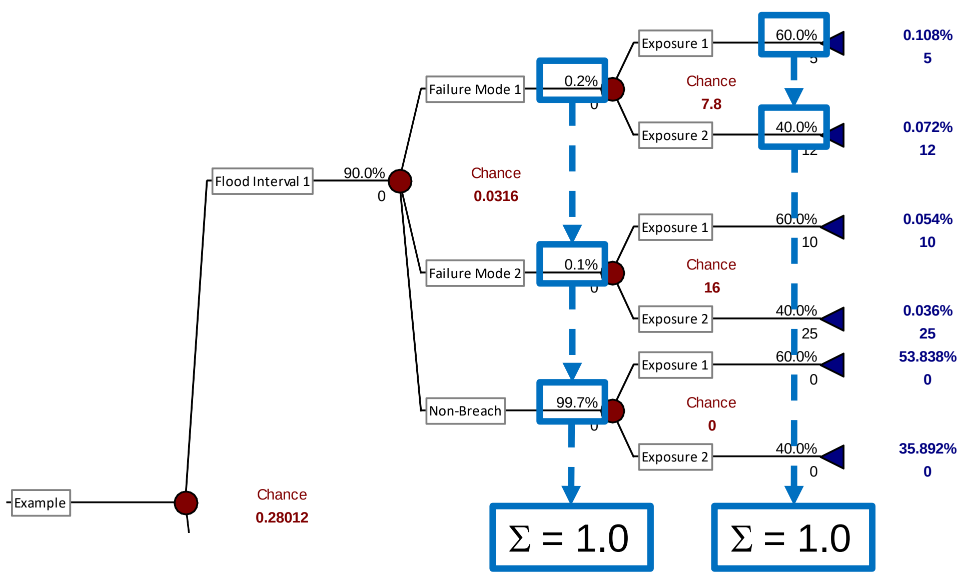


Figure A-5-3 Mutually Exclusive and Collectively Exhaustive Concepts

Branch probabilities within a particular level of the event tree can be summed to obtain an aggregate probability (or risk) associated with a set of related events. The event tree in **Figure A-5-4** illustrates the summation of potential failure mode pathway probabilities to obtain the total probability of failure for Flood Interval 1. Total annualized life loss can be similarly obtained by multiplying the failure probability and associated consequences for each end branch and then summing across the end branches.

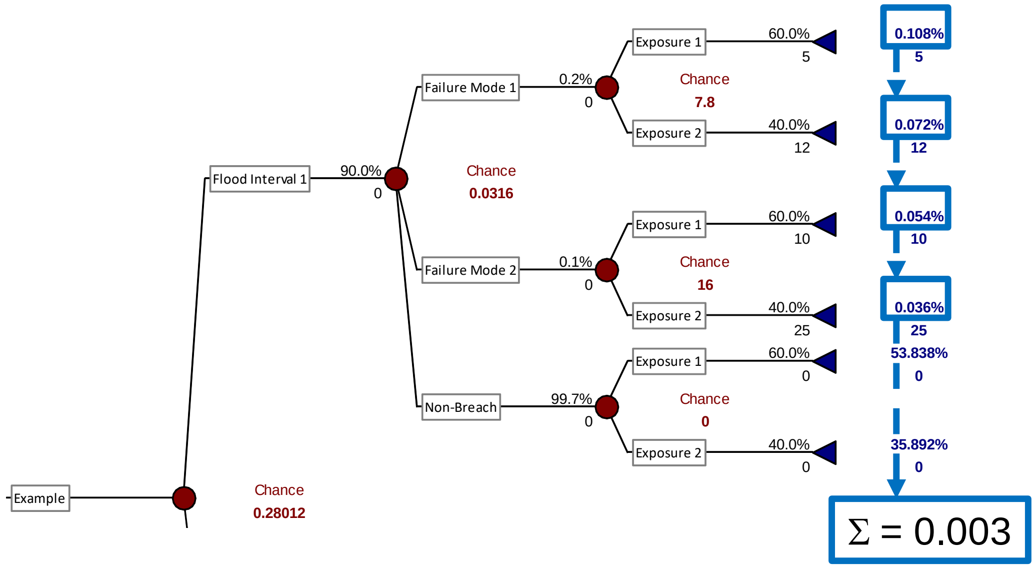


Figure A-5-4 Example Calculation for Probability of Failure

A-5.4 Potential Failure Mode Event Trees

It is common practice to develop detailed event trees for individual potential failure modes to clearly identify the full sequence of steps required to obtain failure or breach. Each identified potential failure mode is decomposed into a sequence of component events and conditions that must occur for there to be a failure. This ensures that due consideration is given to each event in the failure sequence. It also supports the identification of key issues contributing to the risk. An example event tree structure for an internal erosion potential failure mode is illustrated in **Figure A-5-5**. A challenge with estimating probabilities for detailed event trees is remembering that the probability of each event is conditional on that of predecessor events. For the internal erosion event tree example, this means that the probability estimate for the continuation branch should be based on an assumption that the flaw already exists and initiation has already occurred even if the probabilities for a flaw and initiation are very small. Examples and suggested event tree structures for common potential failure modes are provided throughout this manual. The suggested event trees should be adjusted as needed to address site specific conditions.

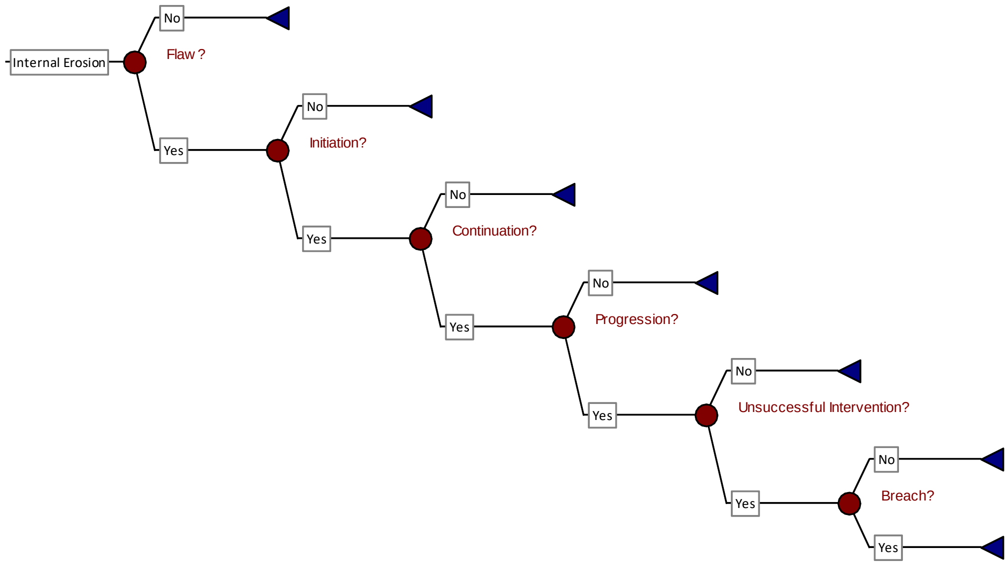


Figure A-5-5 Example Internal Erosion Potential Failure Mode Event Tree

A-5.5   System Response Curves

When system response is evaluated over a range of loads, the resulting relationship is called a system response curve. Other terms describing this relationship, such as a ‘fragility curve’, may be found in other literature. The term ‘fragility’ is intentionally not used in this manual for dam and levee safety due to the negative connotation that results from referring to a dam or levee as being ‘fragile’.

System response probabilities are often elicited separately for different loading events. It is always good practice to plot the resulting system response curve over the full range of load magnitudes to make sure the elicited probabilities and the shape of the system response curve make sense.

Potential failure mode event trees can be used to develop system response curves that describe the probability of failure or breach as a function of one or more loading parameters such as peak water surface elevation or peak ground acceleration. The failure mode event tree can then be evaluated for multiple loading partitions. Branch probabilities that are dependent on the magnitude of the load are modified for each loading partition. These probabilities can be estimated for the nodes of a potential failure mode event tree using a combination of analytical, empirical, and subjective methods. The number and spacing of load scenarios should be sufficient to describe the shape of the system response curve over the full spectrum of potential loads paying careful attention to transitions in system behavior. It is important to identify and include inflection points in the system response curve that represent significant changes in system behavior. A curve can be fit to the resulting data from each of the potential failure mode event trees so that the probability of failure can be estimated for any load condition by interpolation. An example system response curve is presented in **Figure A-5-6**.

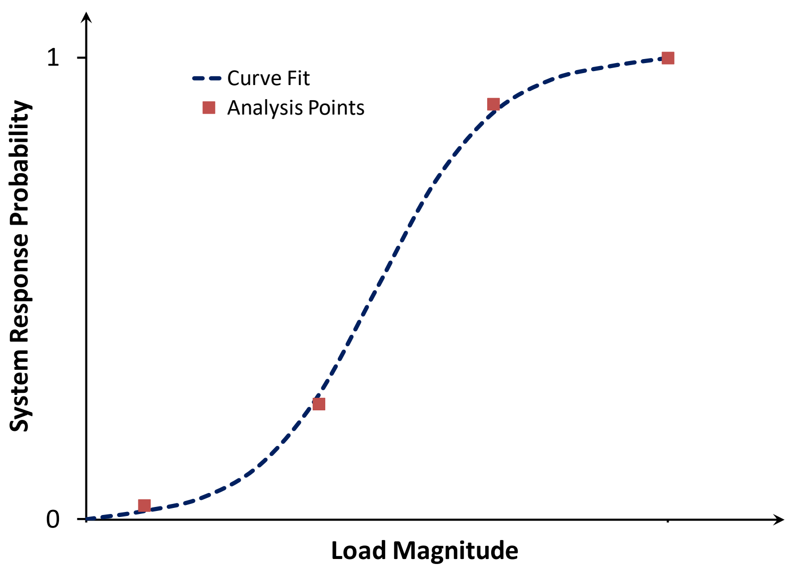


Figure A-5-6 Example System Response Curve

A-5.6 Consequence Event Trees

Detailed event trees can be developed for consequence scenarios. These event trees typically include exposure scenarios to clearly identify the conditions that could lead to different consequences. This ensures that due consideration is given to the factors that may influence consequences. It also supports the identification of the key issues contributing to the estimated life loss. An example event tree structure for exposure is illustrated in **Figure A-5-7**. The example structure proceeds from the longest exposure case (season) toward the left of the tree to the shortest exposure case (time of day) toward the right of the tree so that the overall size of the event tree is minimized. This example reflects the estimation of non-breach consequences. The structure of the example event tree would also apply to the evaluation of breach consequences.

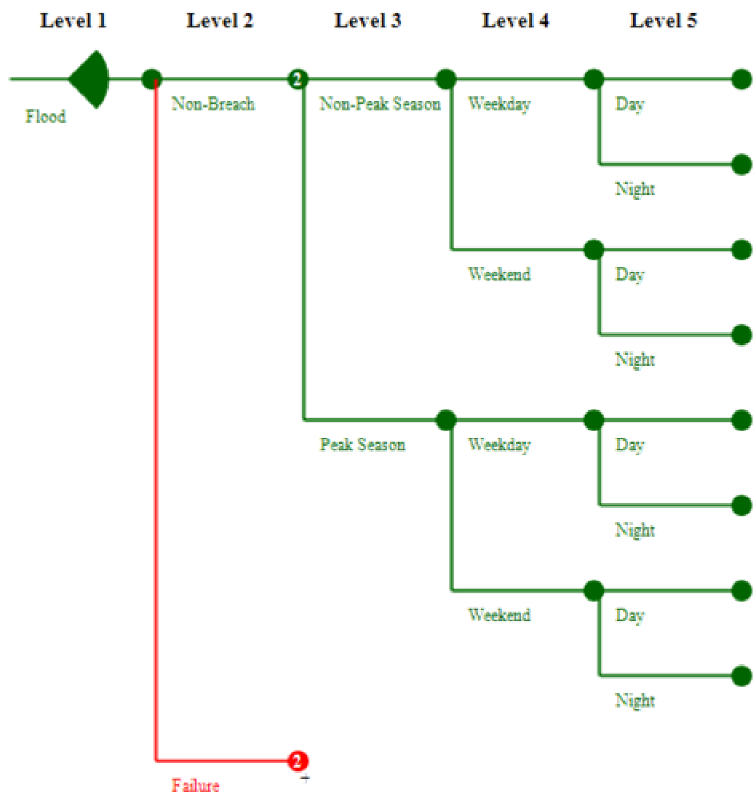


Figure A-5-7 Example Consequence Event Tree

A-5.7 Event Tree Construction

Each branch in the event tree should be clearly defined and representative of a specific event or state of nature. Parallel components can be aggregated into a single event if different combinations are inconsequential to the risk analysis. For example, it might be sufficient to represent failure of a single spillway gate using a single event tree branch if it doesn't matter which particular spillway gate fails. Events that could influence other system components should generally be toward the left of the event tree to reduce the overall tree size. Constructing the event tree in chronological order is not required mathematically, but it usually improves the logic which can facilitate understanding and communication. The event tree structure should be designed to accommodate future needs such as the evaluation of risk reduction alternatives to minimize duplication of effort and provide consistency in the risk estimates across multiple phases of study. Avoid detailed development of branches that do not lead to outcomes important to the risk estimate or risk management decisions. Care should also be taken to avoid situations where the estimated risk becomes a function of the number of branches in the event tree (i.e. adding more branches to obtain a lower risk estimate). Multiple branch levels can be combined into a single branch level when the added resolution does not significantly improve the understanding, estimation, or portrayal of risks. In a multiple branch event tree, event sequences with relatively low probabilities will not control the total AFP and can usually be excluded from the event tree. However, care should be taken to avoid underestimating the risk if too many branches are excluded or if the excluded branches could potentially be significant risk contributors under a different loading condition. Care should also be taken because excluding events can affect the mutually exclusive and collectively exhaustive requirements.

A-5.8 Load Partitioning

In dam and levee risk analysis, the annual failure probability (AFP) can be calculated as the integral of an annual exceedance probability (AEP) versus probability of breach curve. This curve can be obtained by combining (i.e. convolution) a loading curve and a system response curve. Similarly, the annual life loss can be calculated as the integral of an annual exceedance probability (AEP) versus life loss curve obtained by combining a loading curve, system response curve, and life loss curve. The concept is shown in **Figure A-5-8**.

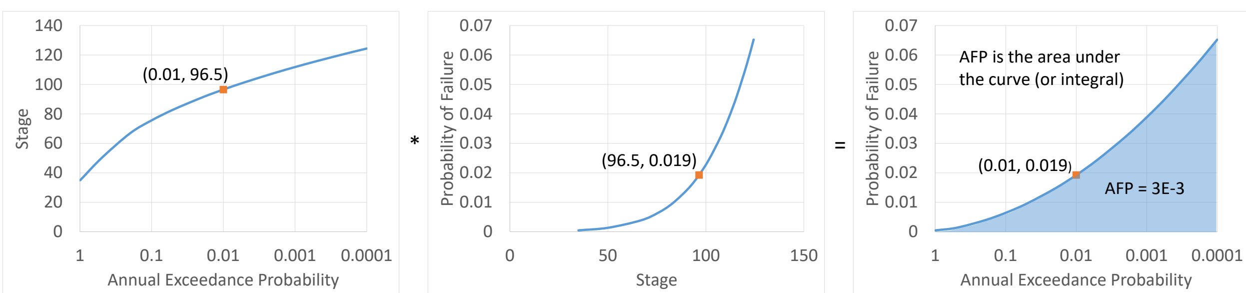
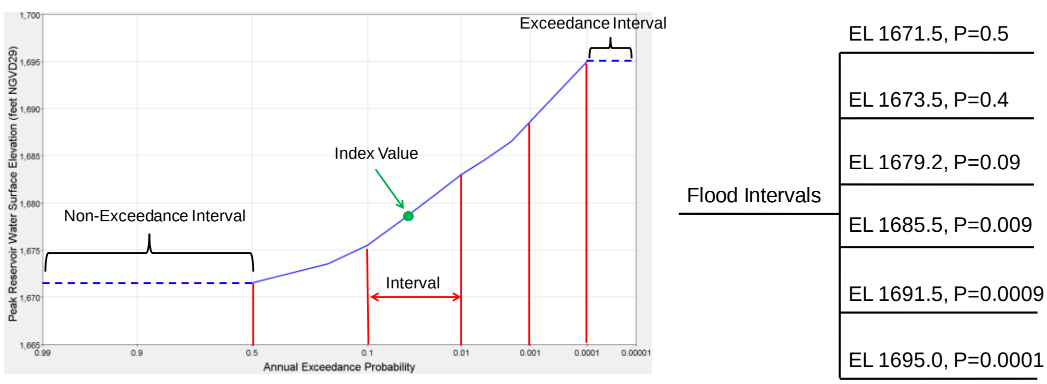


Figure A-5-8 Calculation of Annual Failure Probability

In practice, the discrete branches in an event tree provide an approximation of the integrals needed to calculate annual failure probability (AFP) and annual life loss (ALL). This is analogous to using Simpson's rule (or the Trapezoidal Rule) to calculate a discrete approximation for a definite integral. This is done by dividing the flood or seismic loading curve into discrete loading partitions. The probability for a load interval can be computed from the exceedance probability loading curve as the difference between the exceedance probabilities at the upper and lower bound of the interval. An index value can be estimated for each loading interval. The index value should be a representative or average value over the loading interval. For flood and seismic loading intervals, the geometric mean (obtained by taking the square root of the product of the upper and lower bound) is commonly used as a reasonable average value because these random variables tend to be approximately lognormally distributed. The load partitioning concept is illustrated in **Figure A-5-9**.



Elevation			Probability		
Lower Bound	Upper Bound	Index Value	Lower Bound	Upper Bound	Probability
n/a	1671.5	1671.5	1	0.5	0.5
1671.5	1675.5	1673.5	0.5	0.1	0.4
1675.5	1683.0	1679.2	0.1	0.01	0.09
1683.0	1688.0	1685.5	0.01	0.001	0.009
1688.0	1695.0	1691.5	0.001	0.0001	0.0009
1695.0	n/a	1695.0	0.0001	0	0.0001

Figure A-5-9 Load Partitioning Concept

Index points can be used as reference values in subsequent event tree branches to help estimate the probability of failure and consequences for each interval. The number and spacing of the intervals affects the numerical precision of the risk estimate. The objective is to define enough intervals at a spacing that adequately characterizes the differences in the various event tree input functions. More intervals will improve numerical precision, but can increase the event tree size and estimation burden. When a large number of intervals are used, end branches can be aggregated into bins by summation to facilitate interpretation and communication of results. An examination of event tree probabilities such as probability of failure and other values such as consequences at both the lower and upper bounds of each interval can provide insights as to whether or not the intervals are appropriately sized. A significant change from the lower to the upper bound might indicate a need for more intervals.

The partitions should also consider a non-exceedance and an exceedance interval. The non-exceedance interval can be established based on a threshold loading below which the probability of failure and consequences are negligible. This becomes the bottom end of the lowest load range for which risks are estimated. The lower bound for the non-exceedance interval should be an annual exceedance probability of 1 and the upper bound of the interval should be defined by the threshold event. While simple in concept, the selected threshold value can have a significant influence on the estimated risks. Sensitivity analysis is sometimes performed to evaluate whether refinement of the selected threshold is needed. The exceedance interval establishes the largest loading condition for which risks are estimated. It is important to assess whether or not there could be any significant contributions to the total risks attributable to extreme loading. Would the risk significantly change if an additional higher loading interval was added to the analysis? If agency protocol establishes an upper bound for loading (e.g. probable maximum flood), then the exceedance interval can be defined based on agency protocol. The lower bound for the exceedance interval is the threshold for the largest loading that will be considered and the upper bound of the interval is undefined (open ended interval).

A-5.9 Uncertainty

Risk estimates should give due consideration to uncertainty and sensitivity. Two important questions to consider when evaluating and communicating uncertainty are:

- Does the uncertainty significantly impact the decision?
- How easily can the uncertainty be reduced?

Key areas of uncertainty and sensitivity should be identified and portrayed. This can be accomplished using a variety of qualitative and quantitative techniques. Most of the techniques used in event tree analysis are quantitative.

Sensitivity analysis can be used to evaluate how different assumptions influence the risk estimate. It provides a way to evaluate alternative outcomes if a situation turns out to be different than anticipated. Evaluation of the alternative outcomes can facilitate the identification of key assumptions and the potential value added by additional study. For example, investigations to assess the slip rate for a fault may not be justified if the total risk estimate is not sensitive to the probability estimates associated with this parameter or if the uncertainty in the total risk estimate will not be meaningfully reduced. Reasonable best case and reasonable worst case assumptions can be used as a starting point for the sensitivity analysis. Sensitivity analysis typically only considers a limited number of parameters for each scenario. Combining worst (or best) case assumptions for all input parameters is not recommended because the probability that all parameters are unfavorable (or favorable) in situ is usually remote.

Uncertainty analysis can be accomplished by using probability distributions to define event tree variables. This is done to characterize the knowledge uncertainty in the event tree inputs. This uncertainty can, at least in theory, be reduced by acquiring more information. Distributions that are commonly used in dam and levee safety risk analysis include the uniform, triangular, normal, log-normal, and PERT. The distributions can be applied to variables such as peak reservoir stage, peak ground acceleration, the conditional probabilities of various events, and life loss consequences. Monte Carlo simulation techniques can be used to generate random samples from these distributions. Probabilities and risks are calculated for many random samples (typically 10,000 or more) to characterize the uncertainty in the total risk estimate. It is not always practical or possible to quantify all sources of uncertainty; therefore, it is important to document the significant uncertainties that were included in the event tree analysis and those that were not quantified but are still relevant.

Natural variability (aleatory uncertainty) associated with random events such as floods or earthquakes is typically accounted for in the event tree by the hazard curve itself which defines the full range of plausible event magnitudes and frequencies. The knowledge uncertainty is typically defined using confidence limits around the hazard curve. Both sources of uncertainty in the hazard can be modeled and combined in an event tree analysis. The natural variability is typically modeled by using loading partitions over the full range of loading. The knowledge uncertainty can be modeled by sampling from the uncertainty distribution for each partition. An alternative technique would be to develop and run a stochastic simulation model that randomly selects a flood or earthquake event for each trial of the simulation. Many trials can be performed to capture the full range of possible loading events and outcomes.

A-5.10 Monte Carlo Simulation

Monte Carlo simulation is a mathematical technique that is used to evaluate uncertainty in risk analysis. It provides a means to evaluate the uncertainty in a risk estimate by combining the uncertainties for all of the event tree inputs. Commercial software such as @Risk, Crystal Ball, and many others can be used to perform the calculations. A Monte Carlo simulation samples a possible value for each random variable in an event tree based on its probability distribution. The inverse transform sampling methods is commonly used. A pseudo random number generator (PRNG) generates a random number between between 0 and 1 from the uniform distribution. This random number is then applied as a value of the cumulative distribution function (CDF) for the random variable. A random value for the variable is then obtained by applying the CDF value to the probability distribution for the random variable. When enough samples are taken, the resulting distribution of the random variable samples will very closely match the probability distribution from which the values were sampled.

Each sample in the Monte Carlo simulation produces a single estimate of the total risk for a given PFM. The sampling process is repeated many times to obtain many estimates of the total risk. If enough samples are taken, the compiled estimates of risk can be used to portray a probability distribution of the total risk. It is not uncommon in dam and levee safety risk analysis that 10,000 or more samples are required to obtain a reasonable result. The sampled estimates of risk can be portrayed in various ways to assess and communicate the uncertainty in the total risk estimate. A common approach is to show each sampled risk estimate (FFP or ALL) as a point on the f-N chart (along with the mean value of the risk samples). The resulting cloud of points can provide risk analysts and decision makers with information on the magnitude of uncertainty.

Percentiles can also be estimated from the sampled risk estimates to characterize the likelihood that the total risk estimate will fall either above or below risk guidelines.

Consider the following example for a potential failure mode that could initiate at flood loadings greater than a 10 year flood. The conditional probability of failure was estimated by expert elicitation and the uncertainty in the estimate is described by a triangular distribution with a lower bound of 0.00001, an upper bound of 0.0005, and a most likely (mode) of 0.0002. Potential life loss was estimated from an analytical model and the uncertainty in the estimate is described by a triangular distribution with a lower bound of 60, an upper bound of 120, and a most likely (mode) of 80. An event tree is illustrated in **Figure A-5-10**. PrecisionTree and @Risk were used to develop the event tree and perform the Monte Carlo simulation. The PrecisionTree model settings for @Risk simulation were set to 'expected values of the model' as illustrated in **Figure A-5-11**. The conditional probability of failure and consequences were input using the 'define distribution' feature in @Risk. The annual probability of failure (e.g. value of 'f' for the f-N chart) is equal to the total probability along the breach pathway obtained by multiplying the probability of the flood by the probability of failure. The value of 'N' for the f-N chart is simply the life loss associated with a breach.

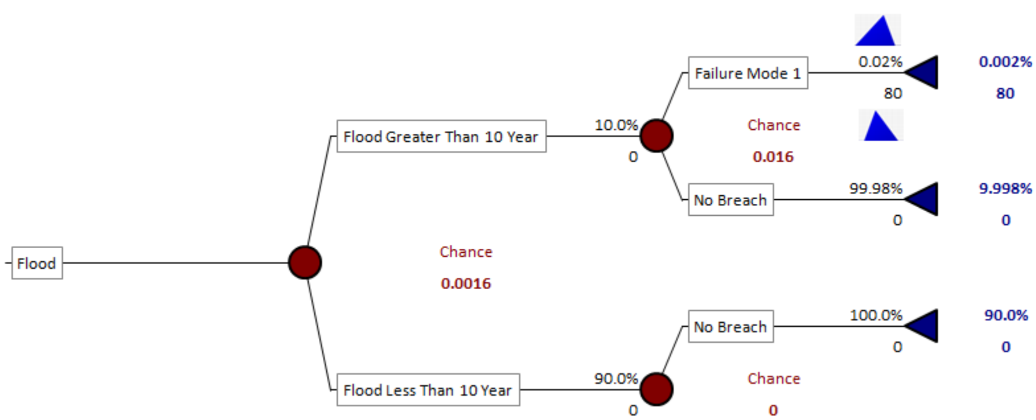


Figure A-5-10 Event Tree for Monte Carlo Simulation Example

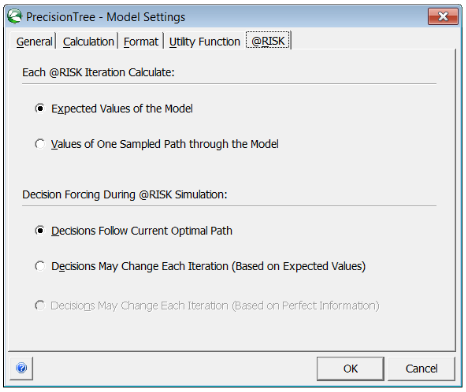


Figure A-5-11 Precision Tree Settings for Monte Carlo Simulation Example

A 1000 iteration Monte Carlo simulation was performed on the event tree using @Risk. The results are presented on the f-N chart in **Figure A-5-12**. The mean estimate was obtained by taking the arithmetic averages of the annual failure probability and life loss samples from the Monte Carlo simulation. Although the Monte Carlo simulation results straddle the risk guidelines, the mean estimate is above the guidelines. By counting the number of Monte Carlo simulation results that are above the guidelines (873) and dividing by the total number of trials (1000), the risk analyst might conclude that there is a relatively high likelihood (about 87%) that the risk actually does exceed the guidelines.

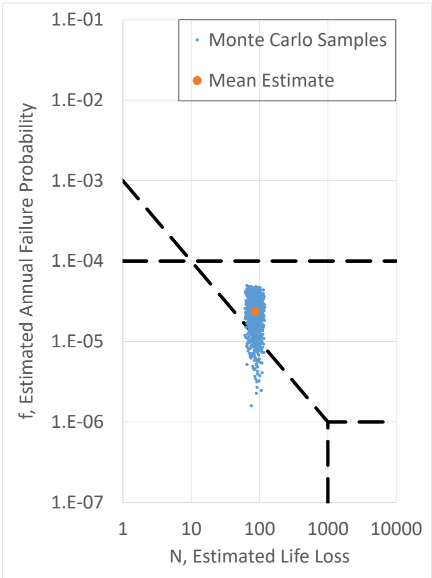


Figure A-5-12 Monte Carlo Simulation Results

A-5.11 Curve Sampling

Typical Monte Carlo simulations include the sampling of values from functions either explicitly or implicitly. This includes sampling of flood and seismic loadings from hazard curves, sampling performance from system response curves, and sampling consequences. It is important that the sampling technique produces numbers in the event tree that are logical and mathematically valid. Consider the simple event tree shown in **Figure A-5-13** with three flood loading partitions in the context of one potential failure mode. The probability of failure for the two breach branches is defined by the triangular distributions shown in **Figure A-5-14**.

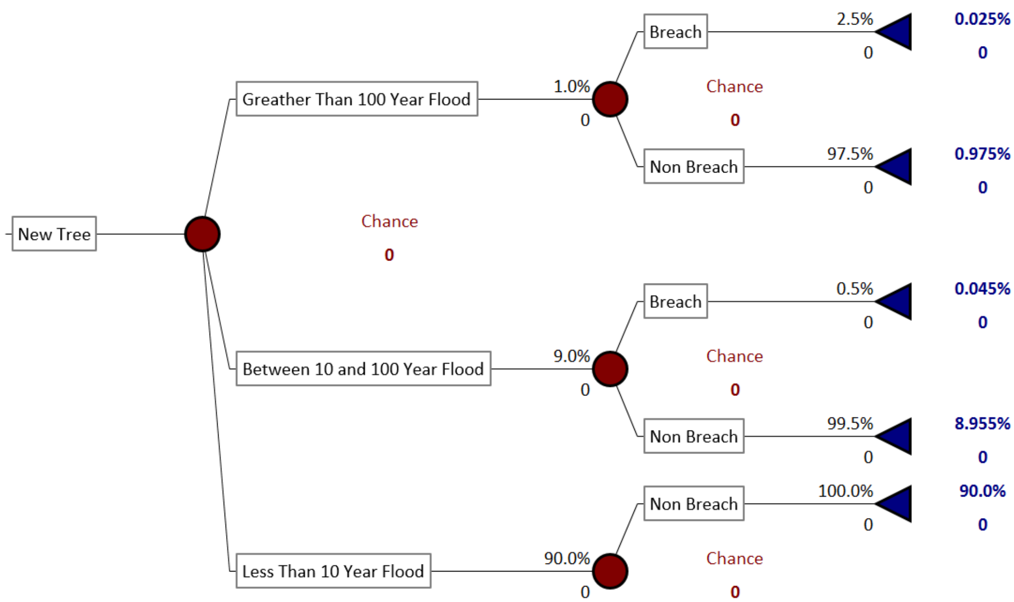


Figure A-5-13 Event Tree for Curve Sampling Example

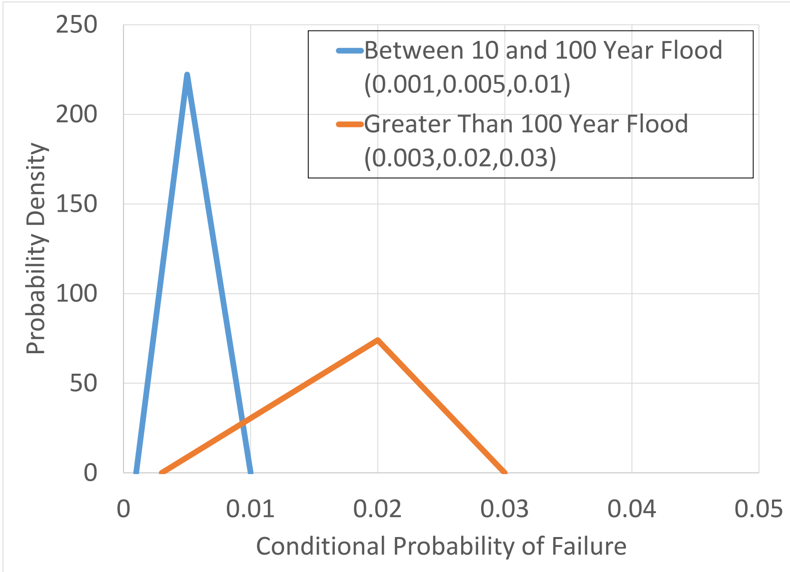


Figure A-5-14 Uncertainty for Curve Sampling Example

If the probabilities of failure for each load partition are independently sampled, the probability of failure sampled for the smaller load partition will sometimes be greater than the probability of failure sampled for the larger load partition. This occurs because of the overlap in the uncertainty distributions over the two loading partitions. This result is physically impossible and mathematically invalid because the probability of failure in this example must increase as the load increases. The higher load partition cannot have a smaller probability of failure. The end result is that the uncertainty portrayed by the Monte Carlo simulation results may not be correct and could potentially misinform decision makers. For this example, a summary of the results for 1000 samples is shown in **Figure A-5-15**. All of the samples should plot below the line of equity. In other words, the sampled probability of failure for the 100 year load partition should always be greater than the sampled probability of failure for the 10 to 100 year load partition. In this example, some of the samples plot above the line of equity indicating an error in the event tree model. Since there are relatively few samples that plot above the line of equity, the error might not be significant in this example.

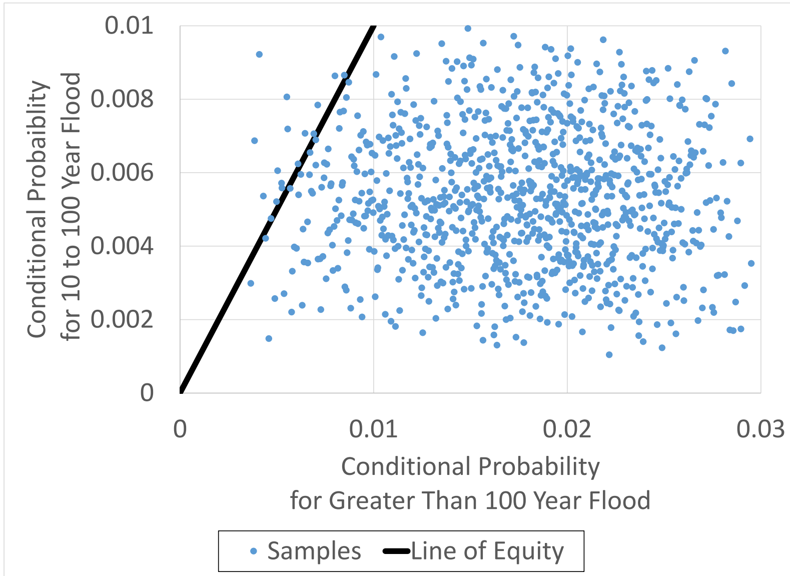


Figure A-5-15 Curve Sampling Using Independent Samples

Similar situations can occur with random variables for other event tree branches. For example, a peak ground acceleration that is sampled for a less frequent load partition could end up being smaller than the peak ground acceleration that is sampled for a more frequent load partition. Again, this would be a physically impossible and mathematically invalid result for that particular sample. PGA must always increase with increasing return period (or with decreasing exceedance probability).

These types of sampling issues can be resolved by generating a sample for the entire curve rather than independently sampling individual load partitions. The consistent percentile method is one technique that can be used for curve sampling. A random number between 0 and 1 is first sampled from a uniform distribution. This value is then applied as a percentile for each of the load partitions. Using the system response from the previous example (**Figure A-5-14**), the concept is illustrated in **Figure A-5-16**. A generated random number of 0.6 would be applied to the cumulative distribution function of system response for both load partitions to obtain probability of failure estimates of about 0.006 and 0.02. The results for 1000 samples are shown in **Figure A-5-17**. Note that all of the samples now plot below the line of equity which means that the probability of failure for the larger load partition is always greater than the probability of failure for the smaller load partition. However, the results reveal a potential disadvantage of the consistent percentile method. The high degree of correlation observed in the samples (**Figure A-5-17**) may not be realistic and may not adequately capture the uncertainty in the risk estimate. More advanced techniques, such as bootstrap sampling, are available to avoid this issue.

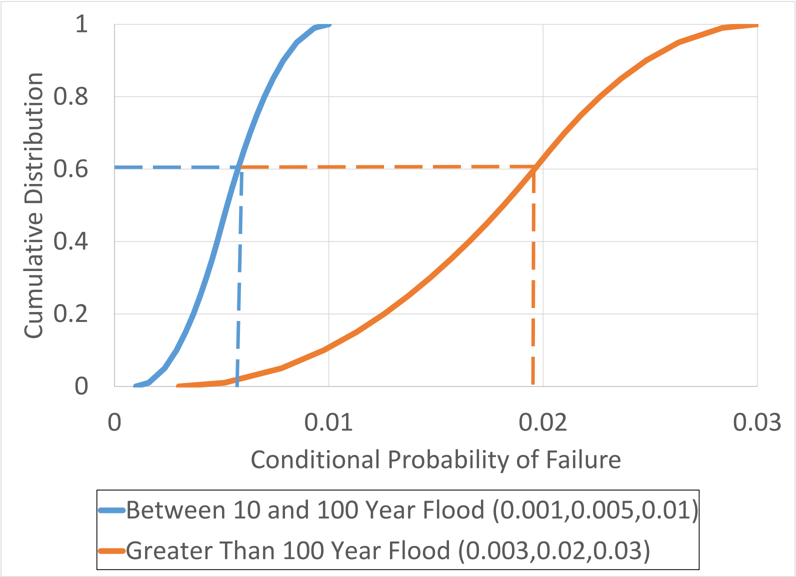


Figure A-5-16 Consistent Percentile Sampling Concept

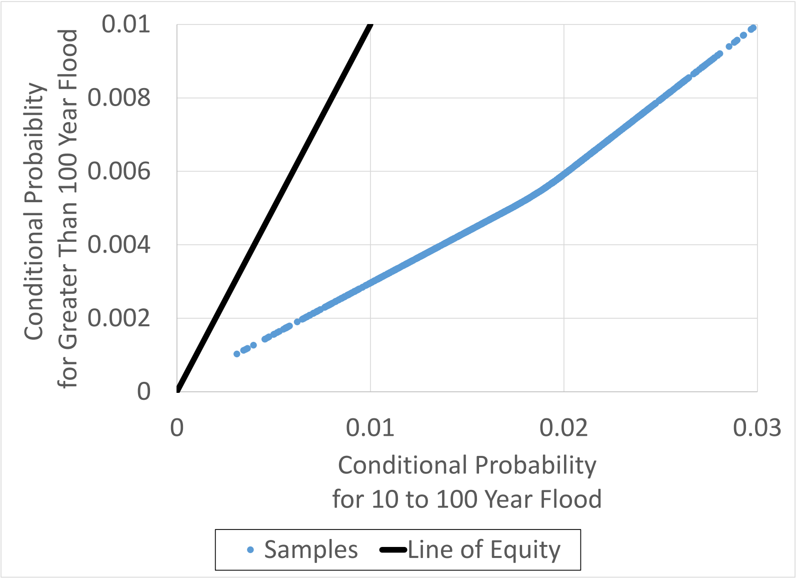


Figure A-5-17 Curve Sampling Using Consistent Percentile

Consider an example where the mean seismic hazard is described by the relationship in **Figure A-5-18**. The uncertainty is characterized by the 5^{th}, 16^{th}, 50^{th}, 84^{th}, and 95^{th} percentile curves.

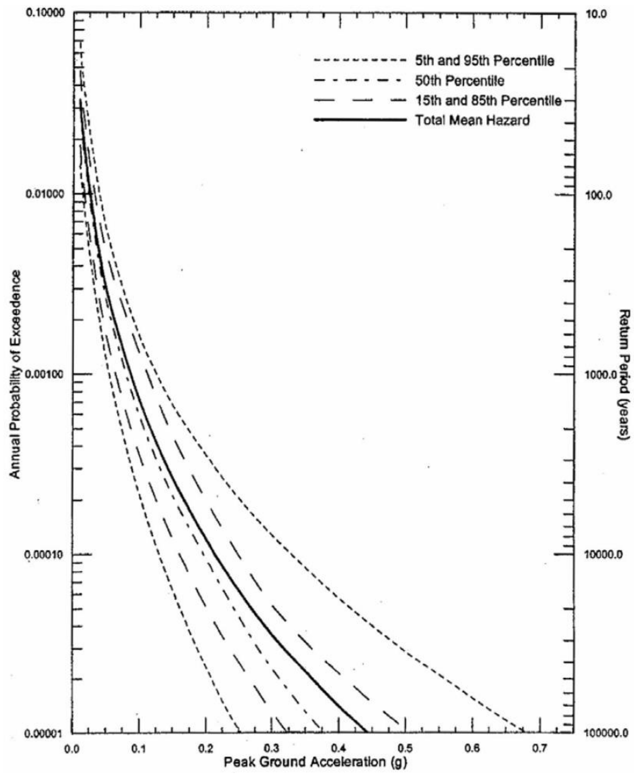


Figure A-5-18 Seismic Hazard Curve Example

The risk analyst has decided to partition the loading for the event tree with two partitions having a peak ground acceleration of 0.08g and 0.2g. The annual exceedance probability values can be tabulated for each percentile and each partition. The concept is illustrated in **Figure A-5-19** for a peak ground acceleration of 0.2g. The results for both load partitions are summarized in Table A-5-1.

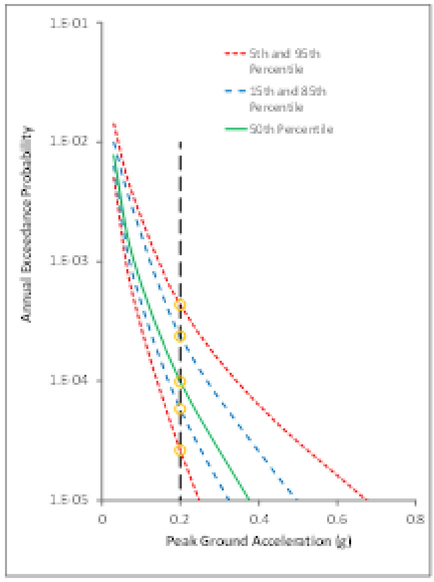


Figure A-5-19 AEP Values for 0.2g Load Partition

Table A-5-1 Summary of AEP Values for Two Load Partitions

PGA	AEP 5th	AEP 16th	AEP 50th	AEP 84th	AEP 95th
0.08	4.90E-04	7.43E-04	1.04E-03	2.63E-03	3.48E-03
0.2	2.62E-05	5.80E-05	9.87E-05	2.36E-04	4.32E-04

The tabulated values can be used to define a probability distribution representing the uncertainty in AEP for each load partition. This can be accomplished by simply using the percentile values from the table and interpolating for intermediate percentile values or by fitting an analytical distribution to the percentile data. The resulting distribution can then be included in an event tree model using Monte Carlo analysis to quantify the uncertainty in the risk. If partitions beyond the limits of the data shown in Figure A-5-17 are needed, the risk analyst must make some assumptions in consultation with the project seismologist to define if and how the data should be extrapolated.

A-5.12 Software

Event tree models and calculations can be prepared using commercial software, custom built spreadsheets, or software specifically designed for dam and levee safety risk analysis. Reclamation typically uses the Decision Tools Suite by Palisade. The Decision Tools Suite includes @Risk for Monte Carlo simulation, Precision Tree for event tree analysis, and several other tools that support statistical analysis. These software packages are fully integrated with Microsoft Excel for ease of use. USACE uses both Decision Tools Suite and a custom software package called DAMRAE (Dam Risk Analysis Engine). The DAMRAE software includes an event tree construction and calculation algorithm specifically designed for dam and levee risk analysis. Other commercial software packages are available.

A-5.13 Exercise

Develop an event tree given the following potential failure mode description.

As a result of high reservoir levels and an increase in uplift pressure on the old shale layer slide plane or a decrease in shearing resistance due to gradual creep on the slide plane, sliding of the buttress initiates. Significant differential movement between adjacent buttresses occurs causing the deck slabs to unseat from their simply supported condition on the corbels. Failure of the upstream slab through two bays results, followed by progressive collapse of the adjacent buttresses due to the lateral loading. An uncontrolled release of the reservoir occurs.